
**EMISSION MEASUREMENT CENTER
TECHNICAL INFORMATION DOCUMENT**

DERIVATION OF ΔH_o

INTRODUCTION

The term ΔH_o given in Equation 1 below is determined during the calibration of the orifice meter in the Environmental Protection Agency (EPA) Method 5 sampling train. The purpose of this document is to explain the derivation of this term.

$$\Delta H_o = \frac{0.0319 \Delta H^f \left[\frac{(t_w + 460)^2}{V_w} \right]^{1/2}}{P_b (t_o + 460)} \quad \text{Eq. 1}$$

DERIVATION

The flow rate through an orifice meter is defined by the following equation:

$$Q_m = K_m \sqrt{\frac{T_m \Delta H}{P_m M_m}} \quad \text{Eq. 2}$$

where:

Q_m = volumetric flow rate through the orifice meter, cfm.

K_m = orifice meter calibration constant, [(in. Hg)(lb-mole)] / [(°R)(in. H₂O)].

T_m = absolute orifice meter temperature, °R.

ΔH^f = orifice meter pressure differential, in. H₂O.

P_m = absolute orifice meter pressure, in. Hg.

M_m = molecular weight of gas flowing through the orifice meter, lb/lb-mole.

Instead of using the orifice meter coefficient K_m , the term ΔH_o was derived to reflect the general sampling conditions of sampling trains and orifice meter designs with a calibration constant that could be related to the desired flow rate. For example, a ΔH_o of 1.84 means that the orifice pressure differential ΔH would be about 1.84 when sample gas is flowing through the orifice meter. The reference conditions for ΔH were chosen to be 0.75 dcfm of air at 68°F and 29.92 in. Hg. Using Equation 2, ΔH_o is

then defined as:

$$) H_{@} = \frac{Q_{@}^2 P_{@} M_{@}}{K_m^2 T_{@}} \quad \text{Eq. 3}$$

where the subscript "@" represents the reference conditions of 0.75 dcfm of air at 68°F and 29.92 in. Hg.

Substituting the values for the reference conditions (except $M_{@}$) into Equation 3, the following is obtained:

$$) H_{@} = \frac{(0.75^2) (29.92) M_{@}}{528 K_m^2} = 0.0319 \frac{M_{@}}{K_m^2} \quad \text{Eq. 4}$$

Now, the relationship between Q_m and the volumetric flow rate through the wet test meter (Q_w) is as follows (the subscript "w" refers to the flow rate conditions at the wet test meter):

$$\frac{P_m Q_m}{T_m} = \frac{P_w Q_w}{T_w} \quad \text{Eq. 5}$$

Substituting Equation 2 into Equation 5 and solving for K_m^2 :

$$\frac{P_m K_m}{T_m} \sqrt{\frac{T_m) H}{P_m M_m}} = \frac{P_w Q_w}{T_w}$$

$$K_m^2 = \frac{M_m T_m P_w^2 Q_w^2}{) H T_w^2 P_m} \quad \text{Eq. 6}$$

In the Method 5 calibration set-up, the pressure at the orifice meter and the wet test meter are at barometric pressure. Thus, $P_w = P_m = P_b$, Equation 6 simplifies to:

$$K_m^2 = \frac{M_m T_m P_b Q_w^2}{) H T_w^2} \quad \text{Eq. 7}$$

Substituting Equation 7 into Equation 4:

$$) H_{@} = \frac{0.0319 M_{@}) H T_w^2}{M_m T_m P_b Q_w^2} \quad \text{Eq. 8}$$

Since air is being used to calibrate the orifice, $M_o = M_m$. In addition, $Q_w = V_w/2$, which is the volume measured by the wet test meter divided by time, and T_m is the same temperature measured at the outlet of the dry gas meter, i.e., $T_m = T_o$. Furthermore, $T_w = t_w + 460$, and $T_o = t_o + 460$. Making these substitutions in Equation 8 yields:

$$)H_o = \frac{0.0319)H \left[(t_w + 460)^2 \right]}{P_b (t_o + 460) \left[\frac{V_w}{2} \right]} \quad \text{Eq. 1}$$

where:

$)H_o =)H$ that gives 0.75 dcfm of air at 68°F and 29.92 in. Hg, in. H_2O .

0.0319^f = constant, $(0.75^2)(29.92)/(68 + 460)$.

$)H^f$ = orifice meter pressure differential, in. H_2O .

t_w = wet test meter temperature, °F.

2^f = time, min.

P_b = barometric pressure, in. Hg.

t_o = outlet temperature of dry gas meter, °F.

V_w = volume measured by the wet test meter, cf.

BIBLIOGRAPHY

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